Hidden Duality and associated instabilities of Tomonaga-Luttinger Liquid on Lattice

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Abstract

Hidden duality and associated instabilities of the spinless Luttinger liquid on lattice are reported. The local quantum fluctuations and the long-distance chiral modes compete and as a result produce a hierarchy of exotic charge/density instabilities. Explicit bosonic quantum operators for the local density fluctuations are constructed and are used to make identification of the Luttinger liquid with the classical 2D Coulomb gas with θ-term and with the rich hidden duality.

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I report novel instabilities and hidden duality structure of the spinless Luttinger liquid on lattice with anomalous *local* quantum fractional statistical fluctuations which, I argue, are induced by the possible multi-particle umklapp and pairing processes at rational densities. The local parameter is, in general, different from the usual charge stiffness K which characterizes the *long-distance* physics. It is shown that the interplay between the *short-* and *long-distance* physics gives rise to new exotic charge/density instabilities which expose the hidden duality of the Luttinger liquid on lattice.

First, I need to consider two conjugate phase fields $\theta(x)$ and $\phi(x)$. The phase $\theta(x)$ essentially defines a field that measures the density modulations, and it acquires phase $\pm 2\pi$ going from one to a neighboring particle. The canonically conjugate phase field $\phi(x)$ is associated with the $U(1)$ charge degrees of freedom such that $[\phi(x), \theta(x')] = i\pi \mathrm{sgn}(x - x')$. Explicit constructions of the two operators are straightforward via Fourier transform. Usually, the renormalization coupling constant $e^{-2\varphi}$ $(= 2\pi K)$ is introduced to code the effects of quantum fluctuation [1].

In the Luttinger liquid universality class it is always possible to find the right and left eigenmodes which carry, in general, fractional statistics. In order to show this more explicitly I use the following right and left Mandelstam modes [2]

$$\Psi_R^\dagger(x) = e^{i\phi(x)} e^{i\beta\theta(x)}; \qquad \Psi_L^\dagger(x) = e^{i\phi(x)} e^{-i\beta\theta(x)}. \tag{1}$$

The time-dependent correlation function for large x and t is given by

$$\langle \Psi_R^\dagger(x,t)\Psi_R(0,0)\rangle \propto \frac{1}{(x - v_s t)^{2x_R}(x + v_s t)^{2x_L}}, \tag{2}$$

where $x_{R,L} = (2\beta\exp(\varphi) \pm \exp(-\varphi))^2/4$. If $2\beta = \exp(-2\varphi)$ then the correlation function involves either the right- or the left-movers only. Therefore, the Mandelstam modes with $2\beta = \exp(-2\varphi)$ can be regarded as the FQS-carrying *long-distance* chiral eigenmodes of the Luttinger liquid.

I conjecture that there are two parameters for describing the Luttinger liquid on lattice. One is the long-distance charge stiffness $K (= \exp(-2\varphi)/2\pi)$ previously discussed, and the other the local FQHE-like parameter which is presumably determined by the filling fraction and *allowed* local interactions such as the umklapp processes.

Now, consider perturbing the Luttinger liquid with the density fluctuations containing the following

$$\Psi_{\lambda,n}^{\dagger m} = e^{in\theta(x)} e^{im\phi(x)} e^{im\lambda\theta(x)}, \tag{3}$$

where m is integer content of the $U(1)$ charge and n integer index for the low-energy sectors. The parameters m, n, and λ are chosen such that the overall operator be bosonic and the resulting damping term equal to constant (allowed umklapp condition). The scaling dimension,

$$x_{n,m}^\lambda = \frac{1}{4\pi K}(n + \lambda m)^2 + \pi K m^2, \tag{4}$$

is invariant under the following duality $\hat{D}$ and periodicity $\hat{T}$ transformations [3]

$$\hat{D} : \eta \to 1/\eta; \qquad (n, m) \to (-m, n), \tag{5}$$
$$\hat{T} : \eta \to \eta + i; \qquad (n, m) \to (n - m, m), \tag{6}$$

where $\hat{D}^2 = 1$ and η is a complex parameter defined as $2\pi K + i\lambda$. This duality generalizes the well-known duality for $\lambda = 0$.

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